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CONVERGENCE BETWEEN BEHAVIOR ANALYSIS AND ECONOMICS ON THE EXPLANATION OF CHOICE BEHAVIOR

CONVERGÊNCIAS ENTRE ANÁLISE DO COMPORTAMENTO E ECONOMIA NA EXPLICAÇÃO DO COMPORTAMENTO DE ESCOLHA

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ABSTRACT

This paper aims to identify how behavior analysis and economics are complementary on the explanation of choice behavior, particularly intertemporal choices, through a literature review. Both behavior analysis and economics researchers have used mathematical models, usually exponential and hyperbolic formulations, to quantify intertemporal choice behavior. Considering theoretical aspects and characteristic models from each area, the present paper concludes that, despite differences in approach, exchange between findings from behavior analysis and economics can contribute to the integrated advance of scientific knowledge.

Keywords: behavioral economics, choice behavior, intertemporal choice, quantification of choice.

RESUMO

Por intermédio de uma revisão da literatura da Análise do Comportamento e da Economia sobre comportamento de escolha, particularmente o de escolhas intertemporais, este artigo busca indicar a complementariedade das duas áreas de conhecimento na explicação de um objeto de estudo comum. Tanto a Análise do Comportamento quanto a Economia têm empregado modelos matemáticos, notadamente em formulações exponenciais e hiperbólicas, para mensurar o comportamento de escolha intertemporal. Partindo de aspectos teóricos e de modelos característicos de cada área, o presente artigo conclui que, a despeito de diferenças na abordagem, o compartilhamento recíproco dos achados da Análise do Comportamento e da Economia pode contribuir para o avanço integrado do conhecimento científico.

Palavras-chave: economia comportamental, comportamento de escolha, escolhas intertemporais, quantificação de escolha.

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In 2002, the Nobel Prize in Economics was awarded to psychologist Daniel Kahneman of Princeton University in recognition of his work on decision-making in situations with probability of gain or loss. Todorov, Coelho, and Hanna (2003) reported that researchers from different areas of psychology were "euphoric" with the award. They also emphasized the importance of the interaction between economics and psychology demonstrated from the economic analysis of decision making and the central role of psychological events in situations involving choice with conflicting alternatives. Indeed, Kahneman's Nobel Prize clearly depicts two current tendencies of the scientific movement: the recognition of psychology and economics as complementary fields of study; and the emphasis on research on decision-making and choice.

The growing rapprochement between psychology and economics can be exemplified by the publication, in 1997, of a special edition of Oxford University's *Quarterly Journal of Economics* (the oldest English-language economic journal) entirely devoted to the overlapping field of behavioral economics. In addition, it is estimated that today 20% of graduate theses in economics at leading American universities (such as Harvard, MIT, Princeton, Yale, and Stanford) are related to that interdisciplinary field (Troyjo, 2007). In 2017, 15 years after Kahneman's Nobel, the prestigious award once again goes to the field, crowning the economist Richard Thaler for his contributions to behavioral economics.

As far as studies on decision-making and choice are concerned, research limits are currently quite broad, including different areas as economics, psychology, philosophy, sociology, anthropology, mathematics, and statistics. Iyengar (2010) points out, in this context, that the concept of "choice" can gain several meanings and its study can be conducted according to several different approaches. A considerable amount of research on decision making is dedicated to better understanding the so-called intertemporal choices, which involve the evaluation of costs and benefits occurring at different times. This type of choice is present in many situations of our daily lives. Loewenstein and Elster (1992) mention, for example, the choice between sleeping late or waking up early; snacking or eating a healthy meal; buying a sports car or a safe sedan; finding a job or attending college; risking pregnancy or using contraceptives.

The psychological processes underlying intertemporal choice have been targeted by studies in both economics (e.g., Loewenstein & Elster, 1992) and behavior analysis (e.g., Logue, 1988; Rachlin, 1989). Applications resulting from research on intertemporal choice are numerous and varied. Meier and Sprenger (2007), for instance, used this tool to explain individual financial behavior (savings and consumption), expanding previous research on behavior patterns in the credit card market (Laibson, Repetto, & Tobacman, 2007; Shui & Ausubel, 2005). Schoenfelder and Hantula (2003) applied the concepts of intertemporal discounts to analyze the career choices of undergraduates. Different applications were used in the studies of Fehr and Zych (2000) and O'Donoghue and Rabin (2002), on the formation of habits and the development of addictive behaviors (Rachlin, 2000).

Based on theoretical aspects and intertemporal choice models, this paper will attempt to demonstrate that, despite the different approaches of behavior analysis and economics (regarding the experimental subjects, the research design, the adopted scientific method, and the techniques applied to data analysis, for example), both areas are complementary. Consequently, sharing research findings in each field may contribute to the integrated advancement of scientific knowledge.

Behavior Analysis Approach

Choice is to respond to one stimulus among two or more available options and preference means spending more time responding to one stimulus (Skinner, 1950) or responding more frequently to one stimulus (Hanna, 1991). Decision making, choice between alternatives, and preference are behaviors that are constantly occurring (Todorov & Hanna, 2005). Even with simple reinforcement schedules, in which only one reinforcement contingency has been programmed, many concurrent responses, and their corresponding reinforcers, are possible beyond those planned by an experimenter (Herrnstein, 1961).

For behavior analysis, choice behavior in itself does not say much: interest resides on the relations between organism and environment that characterize such behavior. In essence, it tries to comprehend where, when, and as a function of which variables are choice and preference altered (Todorov, 1982).

Herrnstein (1961) was a pioneer in the investigation of the relation between distribution of behavior among alternatives and distribution of reinforcing stimuli. Based on these investigations, he formulated the Matching Law, which proposes that relative frequency of responding as well as relative time allocated to an alternative match the relative frequency of obtained reinforcers for that alternative:

$$R_1 / (R_1 + R_2) = S^R{}_1 / (S^R{}_1 + S^R{}_2) \quad (1)$$

$$T_1 / (T_1 + T_2) = S^R{}_1 / (S^R{}_1 + S^R{}_2) \quad (2)$$

in which R, T, and S^R refer to response frequency, allocated time, and obtained reinforcers, respectively, and the numbers indicate the choice alternatives.

Equations 1 and 2, proposed by Herrnstein (1961), gained much interest due to their applications to intermittent reinforcement conditions, when the reinforcing stimulus is presented occasionally and according to rules specified by the schedules of reinforcement (Ferster & Skinner, 1957).

However, in some cases the distribution of responses does not match the distribution of obtained reinforcers (Todorov & Hanna, 2005). In fact, a decrease in behavior sensitivity has been observed: changes in the distribution of obtained reinforcers were followed by smaller changes in the distribution of responses. This distortion usually happens due to specific problems of the experimental procedure, meaning that procedures may incorporate characteristics from natural settings and, therefore, point out the importance of the conditions in which choice occurs.

Operant behavior experiments with pigeons will be taken for analysis. The subjects, before being exposed to the experimental conditions, are trained to eat at the feeder and peck keys in an operant chamber. This training involves some sort of response shaping technique and food is usually used as reinforcement (Keller & Schoenfeld, 1950; Skinner, 1953). During training, usually every peck is reinforced (continuous reinforcement schedule). In the first experimental session with concurrent variable interval schedules, in which responses are reinforced after different intervals of time have elapsed since the last reinforcer, pigeons tend to alternate frequently between keys when responses are not reinforced, since extinction generates behavioral variability (Keller & Schoenfeld, 1950). Instead of independent and concurrent performance, a simple chain of behaviors occurs, and the reinforced sequence tends to be repeated. Once a chain of alternations, or changeovers, has been established, the animal simply no longer discriminates between alternative sources of reinforcement.

To avoid this problem, Herrnstein (1961) programmed a changeover delay (COD) for alternating response patterns: no response would be reinforced before 1.5 s had elapsed since the last changeover. The COD was created as a penalty for changing between schedules and to temporally separate responses emitted under one reinforcement schedule from those emitted under another. When a COD is at least 3 s long, or when there is another consequence for changeover responses that hinders formation of simple chains of behavior (cf. Baum, 1982; Boelens & Kop, 1983; Pliskoff & Fetterman, 1981; Todorov, 1971; Todorov, Acuña-Santaella, & Falcón-Sanguinetti, 1982; Todorov & Souza, 1978; Todorov, Souza, & Bori, 1993), Equation 1 tends to describe well the relation between behavior and its consequences in concurrent variable interval schedules. Even so, Equation 1 has been considered inadequate to explain a fair amount of experimental results. To fit these data, Baum (1974, 1979; Baum & Rachlin, 1969) proposed an equation with additional parameters, known as the generalized matching law:

$$R1 / R2 = k\,(SR1 / SR2)^{a} \qquad (3)$$

in which parameter *k* is a measure of bias, that is, of preference for an alternative, caused by variables other than reinforcer frequency (Cunha, 1988; Todorov & Bigonha, 1982), and parameter *a* is a measure of behavioral sensitivity to the distribution of reinforcers among alternatives (e.g., Hanna, Blackman, & Todorov, 1992; Todorov, Oliveira-Castro, Hanna, Bittencourt de Sá, & Barreto, 1983).

De Villiers e Herrnstein (1976) adapted the matching law equations to a single alternative setting (see also, de Villiers, 1977). This adaptation arises from Herrnstein's (1970) already recognized principle that, even when only one alternative is programmed, response rate reflects a choice setting (e.g., between lever-pressing and scratching, sniffing or any other distraction; Gonçalves, 2005; McDowell, 1988, 1989). The equation that reflects the frequency of behaviors when there is no more than one programmed alternative is

$$R_1 = k.S^R{}_1 / (S^R{}_1 + S^R{}_e) \qquad (4)$$

in which *k* corresponds to the asymptotic response rate in the absence of alternatives and $S^R{}_e$ corresponds to the sum of every other reinforcer except those programmed (De Villiers, 1977; De Villiers & Herrnstein, 1976; Gonçalves, 2005).

In the interface between behavior analysis and economics, the Matching Law was theoretically placed within a context of theory of evolution (Logue, 1988) associated to rational behavior – that is, to maximizing behavior –, following the reasoning that organisms have a better chance of survival when maximizing, through a period of time, obtained reinforcers in a given situation (Rachlin, 1989; Rachlin, Battalio, Kagel, & Green, 1981).

To better explain maximizing, one must introduce the concept of substitutability, which indicates how much an individual is willing to trade one good for another. When goods are perfectly substitutable, the subject is indifferent between acquiring one, another, or any combination of both. The only dimension of interest is the total quantity of goods. When goods are not perfectly substitutable, the economic value of one can only be determined in relation to the other good for which it may be traded. The greater the availability of one good in relation to the other, the lower the subjective value of the acquisition of an additional unit of that good (i.e., the smaller the marginal value of the good). According to Rachlin (1989), if one considers the possibility of adding the value of two goods, a utility function that could express, for an individual, the total value of a basket of two goods is:

$$V_A = k_1\,(Q_1)^s + k_2\,(Q_2)^s \qquad (5)$$

in which V_A is the total value of a basket composed of quantity Q_1 of good 1 plus quantity Q_2 of good 2, k_1 and k_2 are constants representing the contribution of each good to the total value, and the exponent *s* is a measure of substitutability between goods 1 and 2. Note that, if *s* is equal to 1.0, the goods are perfectly substitutable and the total value depends only on the weighted sum of the value of each good. If one had to choose only one of the items, the maximizing solution would be to simply choose the one available in greater amounts. However, if the items are not perfectly substitutable ($s < 1$), choice depends not only on quantity but also on substitutability.

Economic theory also predicts the existence of another equation that represents budget constraint of choice, based on the assumption that there is no infinite availability of goods. Maximization theory predicts that, given a possible set of alternatives represented by the restriction curve, choice will occur at the point in which subjective utility is maximal (see Figure 1).

The relation between matching and maximizing is subject of much debate (*e.g*., Commons, Mazur, Nevin, & Rachlin, 1987): some believe maximizing only occurs in accordance to matching; others believe the opposite is true. As a matter of fact, in most cases, maximizing and matching make the same correct predictions of behavior. Rachlin (1989) approximates the utility function (Equation 5) to the equations used in behavior analysis by swapping variables A_1 and A_2, which indicate the availability of each good, with variables $S^R{}_1$ and $S^R{}_2$, which indicate frequency of reinforcement in period T:

$$V_A = k_1 (S^R{}_1)^s + k_2 (S^R{}_2)^s \qquad (6)$$

In this case, note that exponent *s* from Equation 6 corresponds to exponent *a* from the Generalized Matching Law (Equation 3), and that sensibility to the distribution of reinforcers corresponds to the substitutability measure.

Specific experimental conditions, however, showed that organisms may not be able to maximize reinforcers but will match response distribution to distribution of obtained reinforcers (Herrnstein & Vaughan, 1980). Experimental data (*e.g*., Rodriguez & Logue, 1986; Schneider, 1973; Todorov, 1973) has shown that organisms, in a choice situation, are more sensitive to variations in relative frequency of reinforcers than variations in relative magnitude or relative delay – a principle that was not foreseen by theories stating that individuals tend to distribute their responses to maximize available reinforcers (*e.g*., Logue, 1988; Rachlin et al., 1981; Rachlin, Logue, Gibbon, & Frankel, 1986).

Thus, matching does not correspond perfectly to maximizing. Identifying maximizing using rationality, as is dear to economics, theoretically means that, in specific situations, individuals who match their choices are behaving quasi-rationally. Simon (1978) created the term "satisficing" for situation in which, considering time and information limitations, choice is suboptimal. An example of satisficing would be when individuals don't count their change meticulously when given in coins. One would expect that such individuals will sometimes lose money. However, considering a wide temporal horizon, one may argue that the time and mental effort saved in not counting change compensates for financial loss. Thus, in the long-term, satisficing could also be considered an example of rational behavior.

Rachlin (Rachlin, 1989; Rachlin, Green, & Tormey, 1988) considers that the discussion of the preponderance between matching and maximizing is strictly conceptual, and depends only on how these terms are defined. If maximizing corresponds to a broad interval of time, then it will have difficulty predicting choice behavior. However, if maximizing simply means that choice behavior corresponds to the optimal option in an utility function that refers to a specific time interval, then such behavior can be correctly described as "rational". Matching may be seen as a form of maximizing in a limited future, and as future events are more and more discounted, matching and maximizing tend to converge (for a comparison between molar and molecular views of choice theory, see Baum, 2004). Based on this understanding, Rachlin (1989) elaborated an interesting table (Table 1) showing the equivalence between economic and behavior analysis terminology:

Table 1.
Comparison of Terminology Used in Operant-Choice Experiments and in Economics (Rachlin, 1989).

	Operant Choice	Economic Choice
Objective contingency	Schedule of reinforcement	Constraint
Positive outcomes	Rewards [reinforcers]	Goods – Commodities
Negative outcomes	Punishers	"Bads" - Commodities
Symmetrical choice	Concurrent schedules of reinforcement	Allocation of budget between goods
Asymmetrical choice	Single schedule of reinforcement	Allocation of time between work and leisure
Subjective choice process	Matching	Maximizing

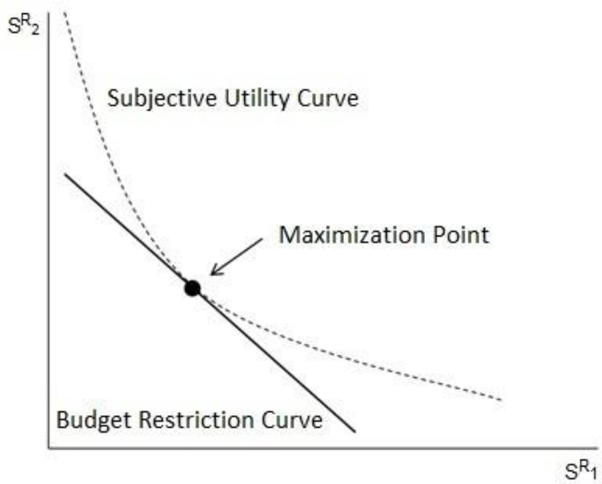

Figure 1. Maximization: choice between goods S^R_1 (x axis) and S^R_2 (y axis), subjective utility curve, budget restriction curve, and maximization point.

In the historical development of research on matching, the logarithmic transform of the matching equation enabled studying other parameters of reinforcing stimuli. The generalized matching law (Equation 4), in its logarithmic form, may be expressed as:

$$\log (R_1 / R_2) = \log k + a \log (S^R_1 / S^R_2) \qquad (7)$$

Neuringer (1967) proposed an extension of Herrnstein's (1961) original equation, in which frequency and magnitude – originally computed as duration of access to food –, of alternative reinforcers vary, using a simple multiplication rule to relate the distribution of responses to distribution of the combined effects of reinforcement frequency and duration:

$$R_1 / R_2 = (S^R_1 \,.\, A_1 / S^R_2 \,.\, A_2) \qquad (8)$$

in which A is the duration of the reinforcing stimulus. Using the log transform of the Matching Law, Schneider (1973) and Todorov (1973) independently showed that, in choice situations where frequency and magnitude of reinforcing stimuli vary, frequency is more important than magnitude:

$$\log (R_1 / R_2) = \log k + a \log (S^R_1 / S^R_2) + b \log (A_1 / A_2) \qquad (9)$$

in which A is the exponent that measures behavioral sensitivity to changes in reinforcer magnitude in terms of duration of access to food for pigeons (Oscós & Todorov, 1978; Todorov, 1973; Todorov, Hanna, & Bittencourt de Sá, 1984) or number of food pellets for rats (Schneider, 1973). In the aforementioned experiments, the exponent in Equation 5 for frequency of reinforcement (*a*) was close to 1.0 and the exponent for magnitude (*b*) was around 0.5.

Besides magnitude and frequency of reinforcing stimuli, another parameter explored by the researchers was delayed reinforcement (cf. Azzi, Fix, Keller, & Rocha e Silva, 1964). In the natural environment, the consequence of a behavior does not always occur immediately after response emission. It is common for a certain time to elapse between de reinforced response and the presentation of the reinforcing stimulus (delayed reinforcement). Chung and Herrnstein (1967) studied the effects of delayed reinforcement in concurrent variable interval schedules and concluded that the matching principle also applied to his experimental data:

$$R_1 / R_2 = [1 / (1 + D_1)] / [1 / (1 + D_2)] \qquad (10)$$

in which D is the time of delay. Williams and Fantino (1978) analyzed data from Chung and Herrnstein (1967) using another equation in its logarithmic form:

$$\log (R_1 / R_2) = \log k + c \log (D_2 / D_1) \qquad (11)$$

in which *c* is behavioral sensitivity to variations in delayed reinforcement. Williams and Fantion's (1978) re-analysis showed that in Chung and Herrnstein's (1967) experiment the value for *c* in Equation 11 was different between shorter and longer delays. Thus, *c* is a variable, not a constant that is independent of the absolute value of delayed reinforcement.

The combination of frequency, magnitude, and delay of the reinforcing stimulus in the same equation with multiple variables has been used with satisfactory results. Logue, Peña-Correal, Rodriguez, and Kabela (1986) suggested the following equation, combining a larger number of variables:

$$\log (R_1 / R_2) = \log k + a \log (SR_1 / SR_2) + b \log (A_1 / A_2) + c \log (D_2 / D_1) \qquad (12)$$

Equation 12 is the broadest form of the Generalized Matching Law (Baum, 1979). When alternative schedules program reinforcers with the same duration and delay, Equation 12 is reduced to Equation 7. When only delays are equal, it is reduced to Equation 9. When magnitudes and frequencies are equal and delays different, Equation 12 is reduced to Equation 11.

Rodriguez and Logue (1986) used another variation to manipulate the duration and delay of reinforcement values, maintaining reinforcer frequency equal and constant:

$$\log (R_1 / R_2) = \log k + b \log (A_1 / A_2) + c \log (D_2 / D_1) \qquad (13)$$

With this equation, the authors found the value of 0.5 for *b* and for *c*; these results were confirmed in a later experiment (Chavarro & Logue, 1988).

The abundance of equations presented indicates the theoretical efforts devoted to capture all relevant variables in choice behavior in an empirically testable expression. A dimension that has shown to be particularly relevant in the understanding of the relation between reinforcement frequency and response frequency is the delay of the delivery of reinforcement from the emission of a behavior. Situations involving delay, together with those in which reinforcement delivery is associated to a probability, have been named risky choice (Green & Myerson, 1996; Kacelnik & Bateson, 1996). Increasing delays or decreasing the probability of occurrence of an event decreases preference for such an event, that is, decreases its value for an organism – delayed or probabilistic events are discounted (Grace, 1999). Discounting is intimately associated to the economic notion of individual discount rates (IDRs), that is, it measures how much a good loses its subjective value as a function of delay to its availability.

In delayed reinforcement, the period between response emission and delivery of the consequence is one of the most studied variables influencing choice distribution in human and non-human animal studies. According to Gonçalves (2005, p. 16), "the effect of delay on choice originated a series of research that formed the body of what is mostly called, in behavior analysis, *self-control*" (our translation).

Gonçalves' (2005) statement, however, requires a short remark. Hanna and Todorov (2002) advise that many researchers have restricted the generality of the self-control phenomenon by overemphasizing the relation between self-control and delayed reinforcement (Logue, 1988). This phenomenon, as the authors rightly express, is much broader. Skinner (1953, 1963, 1974, 1978) pointed out the importance of this topic, stating that individuals will often partially control their own behaviors when responding produces positive as well as negative reinforcement, that is, in situations of conflict. Thus, according to Skinner's notion of self-control, this phenomenon is a contingency with two consequences (positive and negative reinforcement) for the same controlled response (R*c*). Aversive properties are established for self-controlled behavior throughout individual history, and responses that reduce the probability of this behavior can be strengthened. A second, controlling behavior (R*sc*), sometimes called *commitment behavior* (Rachlin & Green, 1972), is part of the contingency, altering the probability of a controlled response through changes produced in R*c* controlling contingencies. Those changes may (a) reduce/increase intensity of eliciting or aversive stimuli; (b) produce/remove discriminative stimuli; (c) modulate motivation by creating establishing operations; (d) make reinforcers/punishers highly likely; or (e) develop behavioral alternatives that do not imply punishment. Skinner considers, thus, many types of self-control. It is important to emphasize that self-control is not an innate characteristic of individuals. According to Hanna and Ribeiro (2005, p. 175), "self-control is often related to personality traits, innate characteristics of individuals, or inner strength that enables control over one's own actions. This use of the concept contrasts with the fact that the same person may present different degrees of self-control in different situations, and show differential degrees of self-control in similar situations, but in different stages of life" (our translation).

The notion of self-control used by Gonçalves (2005), when referring to delayed reinforcement studies, is the one originally developed by Rachlin (1970, 1974, 1976, 1989) and by Rachlin and Green (1972). Two incompatible concurrent operants, R_1 and R_2, occur in the presence of different environmental conditions ($S^D{}_1$ and $S^D{}_2$), producing differential consequences ($S^R{}_1$ and $S^R{}_2$), with a delayed $S^R{}_2$:

$$S^D{}_1 : R_1 \rightarrow S^R{}_1$$

$$S^D{}_2 : R_2 \rightarrow S^R{}_2 \text{ (delayed)}$$

Since delay of the $S^R{}_2$ reduces its reinforcing value, the probability of R_1 occurring is greater than R_2. However, a R*sc* can modify environmental conditions and invert the probabilities of occurrence of R_1 and R_2. Rachlin and his followers define self-control, in this model, as choice or preference for the alternative with a larger later reinforcer; choosing the smaller sooner reinforcer is referred to as impulsivity (Hanna & Todorov, 2002).

Rachlin and Green (1972) conducted the classical study on commitment responses, using a concurrent-chains schedule with pigeons. In this procedure, the initial link (FR 25, keys A and B) led to one of two possible responses, depending on the 25th emitted response. One response to B led to the illumination of a key after time *T*, after which the response R_2 led to a 4-s blackout followed by 4 s of food. One response to A illuminated two keys after time *T*: response R_1 led to 2 s of food followed by a 6-s blackout, and R_2 produced the same result described for key B. Responses to B, thus, meant a commitment response, since only the delayed reinforcement situation becomes available. The authors observed that preference for key B increased as a function of time *T*. This result became known as the Ainslie-Rachlin Model: the subjective value of the reinforcer, that is, its efficacy (Ainslie, 1975), decreases as the moment of reinforcer delivery becomes more distant in time from the moment of choice. Thus, devaluing occurs, with less preference for the delayed reinforcer than for the immediate reinforcer – which, as we have already seen, is called utility discount of deferred goods.

The Ainslie-Rachlin Model represents the subjective value of reinforcers (x-axis in Figure 2) with hyperbolic curves as a function of time elapsed between emission of behavior and obtained reinforcer (y-axis). The value curves for the sooner/smaller and the larger/later reinforcers cross at a certain point (delay value). After this point, the subjective value of the smaller/sooner reinforcer becomes greater than the value of the larger/later reinforcer, producing an inversion, or change, in preference. At exactly this point, choice between both reinforcers is indifferent, so called the *indifference point* (see Figure 2).

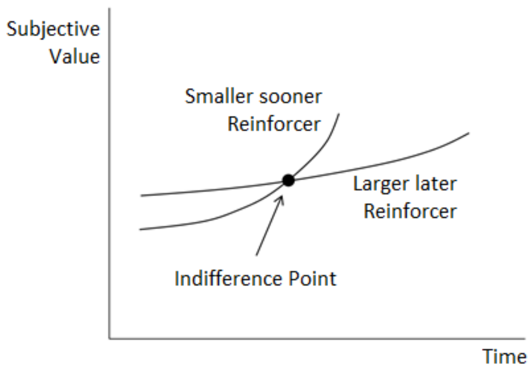

Figure 2. Ainslie-Rachlin model: curves of subjective value of smaller sooner reinforcer and larger later reinforcer as a function of time, and indifference point.

When there are different indifference points for different reinforcer magnitudes, it is possible to graph an indifference curve representing the intertemporal value/discount behavior for a given individual.

The Ainslie-Rachlin Model was expanded for situations involving changes in magnitude and delay of aversive stimuli by Deluty, Whitehouse, Mellitz, and Hineline (1983). In their experiment, reinforcement was substituted by shocks. In an initial period of 5 s, the commitment period, a response led to the presentation of a variable *T* followed by a 5-s delay and 0.5 s of shock. If there were no responses during the commitment period, a choice period occurred after *T*, during which one response immediately led to 0.5 s of shock and absence of responses led to a 5-s delay followed by 5 s of shock. The authors expected a symmetrically opposite behavior to that occurring in the appetitive situation, with preference for the delayed alternative as a function of increase in delay. The results corroborated this hypothesis.

In an attempt to develop a mathematical model to predict influence of delay on the subjective value of reinforcers, Mazur (1987) developed a different procedure from Rachlin and Green's (1972). While Rachlin and Green's study required choice between a commitment alternative and an alternative that led to a new choice between both alternatives, in Mazur's experiment choice occurred for alternatives with different delay values. This was called adjustment procedure or titration schedule.

In this first titration procedure, pigeons were exposed to a contingency requiring an initial response that activated two keys, A and B (Mazur, 1987). Responses to A were followed by a fixed delay and 2 s of access to grain. Responses to B led to an adjustable delay and 6 s of grain. Each experimental session was divided into 16 blocks of 4 trials. In each block, the first two trials were forced choice, one trial in which only key A was illuminated and available for pecking, and another trial in which only key B was illuminated and available. In the other two trials, both keys were illuminated and choice was free. Responses on free trials determined the adjustment of the larger reinforcer: two responses on A decreased the delay by 1 s in the block that followed; two responses on B increased the delay by 1 s; one response on each key maintained the delay.

Note that, with this procedure, Mazur (1987) determined the indifference points, that is, points in which the value of the delayed reinforcer was equivalent to the value of the immediate reinforcer, represented by a similar distribution of responses among both keys. Throughout the experiment, delays from 0 to 20 s were programmed. An indifference point was determined for each delay, from which Mazur evaluated goodness-of-fit of different mathematical values for the relation between the value of a reinforcer and its delay.

The first model presented by Mazur (1987) was the exponential model, in which the reinforcer value is inversely proportional to the delay, in a constant negatively accelerated function:

$$V = Ae^{-KD} \tag{14}$$

in which V is the value or strength of a reinforcer made available after delay D, A represents the value of the reinforcer when made available immediately, K is a parameter representing individual differences that determine how fast V declined with increases in delay, and *e* is the base of a natural logarithm.

The second model presented by Mazur (1987) was the hyperbolic model, in which the value of the reinforcer is also inversely proportional to delay, yet according to a negatively decreasing acceleration function (reduction in subjective value is initially steep and, as delays increase, gradually becomes less steep):

$$V = A / (1 + KD) \tag{15}$$

Finally, Mazur (1987) introduced the hyperbolic-exponential model, in which a parameter S is added to represent individual variation in the evaluation of the delays:

$$V = A / (1 + KD^S) \tag{16}$$

Results obtained by Mazur (1987) favored the hyperbolic model, which fitted well the empirical data. The hyperbolic-exponential model was not discarded, but adding a free parameter, which hinders its interpretation and is less parsimonious, led the author to choose the hyperbolic model as the most adequate.

Mazur (2006) analyzed some peculiarities of choosing an exponential or hyperbolic model. The author states that economists – specifically classic economists, as will be referred to in this paper– favored the exponential equation as an adequate representation of intertemporal discounting since it is apparently more "rational": all the reinforcers are discounted by the same percentage as time passes, independently of magnitude or moment of delivery (that is, IDR is constant). However, as discussed by

Ainslie (1975), if the discounting parameter K (see Equations 14 and 15) is the same for two reinforcers, the immediate and the delayed, the exponential equation does not allow for inversion of preference in a delayed choice situation: a person preferring a larger/later reinforcer today must maintain this preference with the passage of time. However, in specific cases, when K is greater for the smaller than for the larger reinforcer, the exponential equation can predict inversions in preference (Green e Myerson, 1993). The hyperbolic equation, on the other hand, predicts inversion of preference independently of the values set for parameter K (as in the cases, according to Mazur, in which a person who is dieting promises not to get a second serving for lunch, but changes their mind during the meal and eats more than planned).

Harrison, Lau, and Williams (2002) tested the exponential model's assumption that IDR is constant for different delays. To do so, they employed an experimental procedure previously used by Coller and Williams (1999), the multiple price list (MPL). The MPL presents 15 choice binomials between a smaller reinforcer that will be made available in a proximal future date and a larger reinforcer that will be made available on a later date. A different IDR is associated to each choice binomial. Harrison et al. applied the MPL to four different delays: 6, 12, 24, and 36 months.

To control for the presentation of multiple delays affecting participants' responses, two experimental settings were put in place: in one, participants were randomly distributed to a session in which they should consider only one delay; in the other, each participant went through sessions that included all four delays. The authors used questionnaires to collect data on participants' socio-demographic characteristics, financial instruments available to each participant, the annual interest rate for each of these instruments, their current bank balance and individual perception of their possibility of taking on loans, their line of credit, and credit card balance. Correlations were found between some socio-demographic characteristics and participants' IDRs (level of education, professional condition – student, retired, employed, or unemployed –, years after midlife, perception of access to financial instruments).

Harrison, Lau, and Williams' (2002) research showed there is a significant difference between the IDR determined for a 6-month delay and IDRs for the other delays, contradicting the exponential model and corroborating the behavior of the discount rates from the hyperbolic model.

After Mazur (1987) developed and used explanatory mathematical models of the relation between reinforcer value and delay in animal studies, a procedure that became known as delay discounting (DD) was designed for studies with humans. In this procedure, either the smaller/sooner reinforcer or the larger/later reinforcer is fixed and the other reinforcer varies. The goal is to find the pair of values for which the subject is indifferent between choosing one reinforcer or the other. This pair is identified when the subject reverses their preference from one reinforcer to the other, i.e. the switching point. The switching point must be located between the two pairs of alternatives in which reversal occurs. Considering the differences in magnitude between the smaller sooner and the larger later reinforcers on the switching point, it is possible to check the individual discounting rate for a specific delay.

Once the switching points are identified for each delay, it is possible to plot an indifference curve that represents an individual's intertemporal choice behavior. Following Mazur's (1987) rationale, this curve allows us to adjust mathematical equations and evaluate which is a better descriptor of the relation between delay, magnitude, and subjective value of the alternatives presented to human participants (Green, Myerson, Lichtman, Rosen, & Fry, 1996; McKerchar, Green, Myerson, Pickford, Hill, & Stout, 2009; Myerson & Green, 1995). The exponential and hyperbolic models have been tested empirically quite frequently (Gonçalves, 2005).

Myerson, Green, and Warusawitharana (2001) suggested an alternative method to measure discounting of a delayed reinforcer, the area under the curve (AUC). According to these authors, this method does not depend on the mathematical shape of the discounting function and overcomes occasional problems related to the statistics properties of the function's parameters. The AUC is simply the area found under the empirical discount function normalized in a Cartesian plane.

One must also note that behavior analysis has sought to formalize the effects of delay beyond simple reinforcement schedules, with experiments using only concurrent-chains schedules. A concurrent-chains schedule typically involves two schedules in effect during the initial links, each occasionally leading to their own terminal link. Each terminal link has its own reinforcement schedule that finally leads to a primary reinforcer.

Fantino (1981; Fantino, Preston, & Dunn, 1993) formulated a delay-reduction theory (DRT) according to which the value of the conditioned reinforcer in a terminal link in a concurrent-chains schedule is determined by how much a delay is reduced when each terminal link initiates, compared to the mean time for feed presentation before the initial links are in effect. Squires and Fantino (1971) formulated the following equation for DRT:

$$R_1 / R_2 = (S^R{}_1 / S^R{}_2)\ (T_{total} - T_{t1} / T_{total} - T_{t2}) \quad (17)$$

in which $S^R{}_1$ and $S^R{}_2$ are the total reinforcer frequencies, including time of the initial link as well as the terminal link, T_{total} is the total mean time to primary reinforcer since the start of the initial links, and T_{t1} and T_{t2} are the mean times to primary reinforcer from the beginning of the terminal links, that is, mean durations of both terminal links. The DRT has Matching Law as a basic assumption, and reduces to

Herrnstein's (1961) Matching Law when there are no terminal links.

Grace (1994) developed the contextual choice model (CCM), based on the following equation:

$$R_1 / R_2 = (S^R_{i1} / S^R_{i2}) (S^R_{t1} / S^R_{t2})^{(Ti / Tt)} \quad (18)$$

in which R_1 and R_2 are the response frequencies in the initial links of a concurrent-chains schedule, S^R_{i1} and S^R_{i2} are the reinforcement frequencies in the initial links, that is, the frequencies of the onset of each one of the terminal links, and S^R_{t1} and S^R_{t2} are the reinforcement frequencies in both terminal links (frequencies in which terminal links deliver the primary reinforcer). According to the CCM, choice in concurrent-chains schedules depends on the schedules in the initial links inasmuch as the schedules in the terminal links. The CCM is characterized by the Tt/Ti ratio, in which Tt is the mean duration of the terminal links and Ti is the mean duration of the initial links. Since the Tt/Ti ratio is the exponent of the reinforcement rates in the terminal links, the CCM suggests that differences in the terminal links will have greater effects on preference when they are long in relation to the duration of the initial links, and that the terminal links will have lesser effects on preference when they are relatively short.

Grace (1994) derived the CCM from Herrnstein's (1961) basic assumption from the Matching Law: the relative frequency of behavior is proportional to the relative frequency of reinforcement. Grace assumed that terminal link schedules are conditioned reinforcers with values as a function of their reinforcement frequencies (S^R_{t1} and S^R_{t2}). Besides the Matching Law, Grace also took from Baum and Rachlin's (1969) proposal that when reinforcers differ in two or more dimensions (e.g., frequency, delay, magnitude), these factors may be combined multiplicatively to obtain a measure of total reinforcer value. Similarly, Grace argued that reinforcement frequencies in the initial links (S^R_{i1} and S^R_{i2}) may be multiplied by the reinforcement frequencies in the terminal links (S^R_{t1} and S^R_{t2}) to obtain the values of both alternative schedules in a concurrent-chains procedure.

Grace (1994) interpreted that the behavioral expression of the values in the terminal links depends on the context in which they are presented (that is, the durations of the terminal links compared to those of the initial links). Following Baum's (1974) work on the Generalized Matching Law, which has an exponent reflecting behavioral sensitivity to differences in reinforcement rates (parameter *a* in Equation 3), Grace used the Tt/Ti exponent to express the fact that sensitivity to reinforcement rates in terminal links depends on the relative durations of the initial and terminal links. The final result of this set of assumptions was the CCM. Note that Equation 18 is reduced to the simple Matching Law in cases where there is no terminal link (Tt = 0).

As well as the assumption on the crucial role of delayed reduction, the DRT differs from the CCM in its assumption that choice behavior is also a function of total reinforcement frequencies (S^R_1 and S^R_2), compared to the CCM's assumption that it is a function of the reinforcement frequencies of the initial links (S^R_{i1} and S^R_{i2}) (Mazur, 2006).

Mazur (2001) constructed, for concurrent-chains schedules, the hyperbolic value-added (HVA) model, based on three fundamental assumptions: first, as the CCM and the DRT, the HVA adopts the Matching Principle as a basic assumption, reducing to the Matching Law when there are no terminal links; second, the model considers that reinforcer value declines with increases in delay according to a hyperbolic function; third, it predicts that choice depends on increases produced by the value of the conditioned reinforcer, i.e., environmental changes that signal the end of initial link and the beginning of a terminal final. It is noteworthy that Davison (1988) had already adapted the hyperbolic model to concurrent-chains schedules with relative success, but with procedural specificities that compromised generalization. The equation for HVA is:

$$R_1 / R_2 = (S^R_{i1} / S^R_{i2}) (V_{t1} – V_i / V_{t2} – V_i) \quad (19)$$

The two expressions to the left are identical to the CCM. The expression in the parenthesis to the right includes V_{t1} and V_{t2}, the values of the terminal links, and V_i, the value of the initial links. All these values are calculated through a variation of the hyperbolic function (Mazur, 1984).

Empirically, all three models have shown to be adequate predictors of choice behavior. In a comparison conducted by Mazur (2001, 2006), the CCM explained on average 90.8% of the behavioral variability of many non-human animal subjects in concurrent-chains schedules, the HVA explained 89.6%, and the DRT 83.0%. However, the theoretical differences between models must be pointed out. For the CCM, the key factor is the context in which choice is made, more specifically the duration of the time period for choosing. If time for choosing is long in relation to the duration of the terminal links, differences between the schedules in the terminal links will exert relatively little influence on preference. For the DRT, the key factor is delay reduction: preference depends on decreasing time for the reinforcer, which is signaled by the start of a terminal link. Finally, the HVA considers the value of the conditioned reinforcer associated to each schedule as the key variable, and preference depends on increases in the value that is signaled by the start of a terminal link.

Economic Approach

It is worth analyzing, after the exposition of the models used by behavior analysis to describe intertemporal choice behavior, the formalizations used by economics with the same purpose, which are mainly models of discounted utility. Economics, as well as behavior analysis, gives great prominence to the study of intertemporal choice. According to Loewenstein (1992), four distinct historical stages can be identified in the evolution of the economic interpretation of intertemporal choice.

In the first stage, nineteenth-century economists like Senior (1836) and Jevons (1871) explained intertemporal discounting using what today would be called "motivational effects", that is, emotional or hedonistic influences on behavior. Both authors mentioned that the predisposition to temporarily defer gratification would depend on the emotions immediately felt by decision-makers.

In the second stage, marked by the contributions of Böhm-Bawerk (1889, 1914) and Fisher (1930) at the turn of the nineteenth century to the twentieth, intertemporal choice was viewed in cognitive terms as an exchange of present satisfaction for future satisfaction. The reason for the existence of a discount would be the inability of the decision-maker to accurately imagine in the present what the future would look like.

The third stage begins with Samuelson's (1937) formulation of the discounted utility model (DU). Samuelson proposes an equation in which individuals discount future costs and benefits exponentially. A possible utility function that would contemplate the exponential paradigm of choice would be:

$$U = u_t + \sum_{i=1}^{T-t} \delta^i u_{t+i} \qquad (20)$$

This function relates total utility (U) according to the discounted utility (*u*) of activities present in period (t) and of future activities in the periods from t + 1 to T-1. Each individual would present a constant discount factor (δ) for any two periods, i.e., discounts would be the same for immediate choice and for future choice. The individual discount factor (IDF), as an indicator of the time preference of individuals, is inversely proportional to the individual discount rate (IDR), so that:

$$IDF = 1 / (1 + IDR) \qquad (21)$$

Samuelson's model (1937) considers that individuals decide between saving and spending in a perfectly rational way, given their income constraints. According to this view, there is a strong preference for the maintenance of a pattern of constant consumption throughout their life cycles (Deaton, 1992). Saving and spending decisions are made in a way that ensures smooth intertemporal spending, given income differences at different times. These choices, considering diverse individual preferences, are optimal.

The fourth historical stage pointed out by Loewenstein (1992) supersedes the classical model of discounted utility, with the proposition of alternative models that consider the possibility of deviations from perfectly rational behavior. This reaction to the classical model was based on empirical observation. It was found that individual choices hardly fit the exponential function proposed by Samuelson. First, consumption is not smooth over a particular individual's life, but tends to keep up with income variations at different stages of their life cycle (Carroll & Summers, 1991). The greater the current income of an individual, the greater are their immediate expenses, the opposite also being valid. Even in a shorter time span, individuals fail to smooth their consumption. Beneficiaries of social security programs consume more at the beginning of each month and significantly less at the end of the benefit period (Shapiro, 2005; Stephens, 2003). Regular employees likewise convey evidence of heavy spending after receiving their monthly payment, whilst decreasing their level of consumption by the end of the month (Huffman & Barenstein, 2004). Finally, the expectation of predictable revenue (such as tax refunds or bonuses for performance at work) also directly affects spending patterns (Ishikawa & Ueda, 1984; Souleles, 1999). The logical conclusion derived from all these empirical investigations is that individuals do not choose between saving and spending/consuming in a perfectly rational way, i.e., they do not optimize their smoothing consumption, spending a great deal and saving little in certain periods, and saving a lot and barely spending in other periods.

Countless papers have sought to demonstrate that the distortion in the optimization of choice between saving and spending is caused by self-control problems (Benton, Meier, & Sprenger, 2007; Laibson, Repetto, & Tobacman, 2003; O'Donoghue & Rabin, 1999). When planning future choices, individuals optimize their decisions in order to smooth their consumption, but when the same individuals are faced with a present and immediate choice between saving and spending, the relative value of consumption increases as the relative value of saving decreases. Hence most individuals choose the immediate reinforcement of spending, even when the optimal long-term decision would be to save (delayed reinforcement).

As a result of such evidence, research has concluded that, contrary to the traditional economic view of temporal preferences, many individuals do not discount costs and benefits exponentially, but present a bias towards the present (see, for example, Shane, Loewenstein, & O'Donoghue, 2002; Takahashi, 2005). Additionally, evidence suggests that individuals differ substantially as to the degree of their bias to the present (Coller, Harrison, & Rutström, 2005). Temporal preferences biased to the present represent a dynamic inconsistency of choice because it implies that an individual imposes a lower discount factor between now and a future date than between an equal period in the future, similarly to Mazur's critique (2006) of the exponential model.

Differential discounts lead to a problem of self-control (Meier & Sprenger, 2007). Individuals may make plans for choices in future periods, but they will systematically violate these plans by the time these future choices become present (Fisher, 1930; Strotz, 1956). The exponential utility function, elaborated under the premise of optimization of consumption smoothing, is not, therefore, representative of innumerable situations in which individuals act inconsistently. Laibson (1997) and O'Donoghue and Rabin (1999) present an alternative quasi-hyperbolic function that accounts for the differential discounts:

$$U = u_t + \beta\sum_{i=1}^{T-t}\delta^i u_{t+i} \tag{22}$$

In the quasi-hyperbolic model, β represents bias to the present, while δ represents the long-term discount factor. An individual discounts βδ between today and tomorrow, but only deducts between two sequential days in the future at δ. The quasi-hyperbolic model therefore covers cases of dynamic inconsistency in the choices between saving and spending. If $\beta < 1$, individual choice presents dynamic inconsistencies (the present value of consumption is overestimated). If $\beta = 1$, individuals discount exponentially and the quasi-hyperbolic model narrows down to the standard model of exponential discounts. In this case, choices are perfectly rational and optimal in the long run.

Thus, β is also an indicator of the degree of rationality underlying individual decision-making and self-control. The perfectly rational *Homo economicus* ($\beta = 1$) would be able to optimize and maintain his financial planning even when choices become present. In the case of the other economic agents, there would be a bias to the present ($\beta < 1$) and choice would present dynamic inconsistency. The occurrence of dynamic inconsistencies creates the methodological need to apply to research on delay, whenever possible, not an immediate and a delayed reinforcer, but rather two delayed reinforcers, one made available in the near future and another in a more distant future. This procedure, known as front-end delay, aims to neutralize the variation of IDR as a result of bias to the present.

Final remarks

One may note there are some differences between the kind of approach that behavior analysis and economics give to their research. For example, while behavior analysis has a large body of research using non-human animals, economics research is conducted almost exclusively with human participants. The use of non-human subjects by behavior analysis is justified by a concern to maintain rigorous experimental control, a difficult task when participants are human. Experimental control is critical to ensure that data regarding individual behavior (often collected in studies with small n) leads to consistent analysis and results. Economics, roughly speaking, works with greater flexibility in experimental control, compensated by the joint analysis of data from a large number of participants (large n).

The way in which data is collected also differs in each field: behavior analysis uses adjustment procedures, such as titration; economics, for the most part, employs questionnaires. The experimental design is usually different: behavior analysis uses the subject as its own control (within-subject design); economics usually employs group designs. Another difference concerns the scientific method that is applied: behavior analysis prioritizes the inductive method, whilst economics favors the deductive method.

Options regarding type of design and method influence the way each area analyzes obtained data. Behavior analysis prioritizes analyses that highlight each subject in an attempt to understand individual differences. In this context, it uses instruments such as visual inspection and (mainly descriptive) statistical analyses and regressions that focus on the individual. Economics, on the other hand, prioritizes the understanding of the group’s representative relations, with intensive use of econometric techniques and inferential statistics.

These distinctions help explain why behavior analysis and economics have followed different courses for such a long time, with few opportunities for dialogue. The use of distinct vocabulary by scientists from each area, even when dealing with similar issues, hinders interaction. Integration of both fields of knowledge is also hampered by the fact that, in the clear majority of cases, the dissemination of research results from each area is limited to its own verbal community.

Despite such setbacks, however, it is of utmost importance to promote the interface between behavior analysis and economics, insofar as this article has conveyed that information generated by one field of knowledge may complement information obtained in the other. As discussed above, research on decision-making and choice in general and on intertemporal choice in particular are examples of how behavioral and economic approaches are synergistic. Keeping their peculiarities, both behavior analysis and economics have employed mathematical models, notably in exponential and hyperbolic form, in order to measure intertemporal choice behavior.

Integrating results, enabled by the use of the experimental method in both behavior analysis and economics, is not, however, a trivial task. Each of these fields of knowledge generally adopts a distinct stance in explaining and discussing its results. Behavior analysis considers a direct relationship between choice behavior and the environment. Risk aversion, for instance, is seen as a facet of choice in conflicting situations (self-control) that occurs in certain contexts. Economics, on the other hand, generally takes a mediation-type stance (which brings it even closer to cognitive approaches in psychology). Discussion of results in economics thus stems from such a position: in the example of risk aversion, economics understands that this is a self-control problem, consequently making self-control the cause of choice behavior.

Overcoming these obstacles (including the eminently epistemological ones) is of paramount importance to allow the findings of one area to be incorporated into the research of the other towards the integrated advancement of scientific knowledge.

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